KSQUARE CAREER INSTITUTE

SYSTEM OF PARTICLES & ROTATIONAL MOTION

Class 11 Physics • Torque & Equilibrium • NEET / JEE Notes

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Today's Targets

1. Rotational motion of a rigid body
2. Torque (moment of force) — scalar & vector forms, sign convention, net torque
3. PYQs on torque (AIPMT 2009, NEET 2020)
4. Static equilibrium — translational & rotational; seesaw, rods, knife edges
5. Toppling / normal-force shift problems, ladder problems
6. Added: moment of inertia, theorems, angular momentum, rolling — formula sheet

ROTATIONAL MOTION

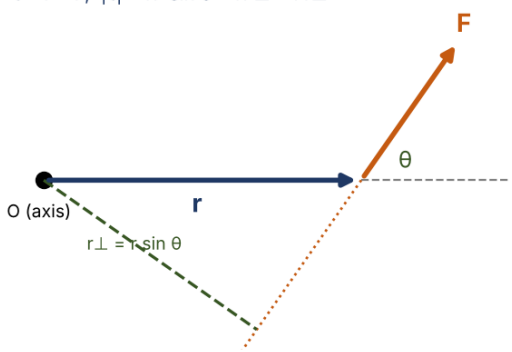
Pure rotation

A rigid body is in **pure rotation** if every particle moves in a **circle** and the centres of all the circles lie on a straight line called the **axis of rotation**.

- **Rigid body:** positions of particles do not change with respect to each other.
- All particles have the **same angular velocity ω**, but **different linear speeds v = rω** (depends on distance from the axis).

TORQUE (MOMENT OF FORCE)

$\tau = r \times F, \ |\tau| = rF \sin\theta = rF_\perp = Fr_\perp$



Torque

$$\tau = r F_\perp = rF \sin\theta \quad (F_\perp = F \sin\theta = \text{component of F perpendicular to r})$$

$$\tau = F r_\perp = F r \sin\theta \quad (r_\perp = \text{perpendicular distance of line of action from axis})$$

$$\text{Vector form: } \tau = r \times F ; \quad |\tau| = rF \sin\theta \ (\theta = \text{angle between r and F})$$

$$\text{Unit: N m}$$

- F cos θ (component **parallel** to r) gives **no torque**.
- If **F ∥ r** (line of action passes through the axis) ⇒ **τ = 0**.
- Direction of τ is **perpendicular to both r and F** (right-hand rule) — along the axis of rotation.

▸ Zero & Non-zero Torque

- Forces through the centre of rotation, or along the rod (F_3, F_4 in the class figure), give **zero torque**; forces perpendicular to the rod at a distance (F_1, F_2) give **non-zero torque**.
- For a block on a surface (about the COM): $\tau_{mg} = 0$ (acts at COM), friction $\tau_f \neq 0$, $\tau_N \neq 0$ when N shifts.

▸ Net Torque & Sign Convention

Net torque

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \ldots$$

$$\text{Anticlockwise (ACW)} \rightarrow +\text{ve} ; \quad \text{clockwise (CW)} \rightarrow -\text{ve}$$

PRACTICE — TORQUE

Q1. If F is the force on a particle with position vector r and τ is the torque about the origin, then **[CBSE AIPMT 2009]**

(a) r·τ = 0 and F·τ = 0
(b) r·τ ≠ 0 and F·τ ≠ 0
(c) r·τ ≠ 0 and F·τ = 0
(d) r·τ ≠ 0 and F × τ = 0

Answer: (a) — τ = r × F is perpendicular to both r and F ⇒ dot products are zero.

Q2. A simple pendulum of length l is pulled aside to angle θ with the vertical. The torque of its weight w about the point of suspension is

(a) wl sin θ
(b) wl cos θ
(c) wl
(d) 0

Answer: (a) — $r_\perp = l \sin\theta$.

Q3. A 1 kg bob on a 1 m string is displaced 30° from the vertical (g = 10). The magnitude of torque about the point of suspension is

(a) 10 N m
(b) 2 N m
(c) 4 N m

(d) 5 N m

Answer: (d) — $\tau = mgl \sin 30° = 10 \times 1 \times ½$.

Q4. Forces of 4 N, 8 N, 9 N and 6 N act on a square/disc arrangement (as in the class figure). The torque about the centre is

(a) 5.4 N m ACW
(b) 1.80 N m CW
(c) 2.0 N m CW
(d) 3.6 N m ACW

Answer: (b) — $\tau_6 = 0$; $\tau_9 = -9/5$; $\tau_4 = +4/5$; $\tau_8 = -4/5 \Rightarrow$ net $= -9/5 = 1.8$ N m clockwise.

Q5. Find the torque about the origin when a force $3\hat{j}$ N acts on a particle at position $2\hat{k}$ m. **[NEET 2020]**

(a) $6\hat{i}$ N m
(b) $6\hat{j}$ N m
(c) $-6\hat{i}$ N m
(d) $6\hat{k}$ N m

Answer: (c) — $\tau = 2\hat{k} \times 3\hat{j} = 6(\hat{k} \times \hat{j}) = -6\hat{i}$.

Q6. The moment of the force $F = 4\hat{i} + 5\hat{j} - 6\hat{k}$ acting at (2, 0, –3) about the point (2, –2, –2) is

(a) $-8\hat{i} - 4\hat{j} - 7\hat{k}$
(b) $-4\hat{i} - \hat{j} - 8\hat{k}$
(c) $-7\hat{i} - 8\hat{j} - 4\hat{k}$
(d) $-7\hat{i} - 4\hat{j} - 8\hat{k}$

Answer: (d) — $r = (2-2)\hat{i} + (0+2)\hat{j} + (-3+2)\hat{k} = 2\hat{j} - \hat{k}$; $\tau = r \times F = \hat{i}(-12 + 5) - \hat{j}(0 + 4) + \hat{k}(0 - 8)$.

STATIC EQUILIBRIUM

Condition	Requirement	Result
Translational equilibrium	$F_{net} = 0$	$a_{COM} = 0$, v_{COM} = constant
Rotational equilibrium	$\tau_{net} = 0$	$\alpha = 0$, ω = constant

Under static equilibrium

$$F_{net} = 0 \text{ and } \tau_{net} = 0$$

Torque about ANY point of the system $= 0 \Rightarrow \tau_{ACW} = \tau_{CW}$

NEET Point / PYQ Alert

Choose the point about which to take torque where the **unknown forces act** — their torque becomes zero and the equation is easy.

Q7. Two kids of 10 kg and 15 kg balance a 5 m seesaw pivoted at the centre. If one sits at an end, where should the other sit?

(a) 2.67 m
(b) 1.98 m
(c) 1.67 m
(d) 1.33 m

Answer: (c) — $10 \times 2.5 = 15 \times x \Rightarrow x = 5/3 = 1.67$ m from the centre (heavier kid).

Q8. Four equal parallel forces act on a rod at 20, 40, 60 and 80 cm from one end (alternately up and down as in the figure). The rod

(a) is at rest
(b) experiences a torque
(c) experiences linear motion
(d) experiences torque and linear motion

Answer: (b) — Net force = 0 but net torque ≠ 0.

Q9. A uniform 20 kg rod hangs horizontally by two threads at its ends and supports a 40 kg mass at l/4 from end A. The tensions are

(a) 300 N, 200 N
(b) 400 N, 200 N
(c) 150 N, 300 N
(d) 100 N, 300 N

Answer: (b) — $T_1 + T_2 = 600$; torque about COM: $T_1(l/2) = 400(l/4) + T_2(l/2) \Rightarrow T_1 - T_2 = 200 \Rightarrow T_1 = 400$ N, $T_2 = 200$ N.

Q10. A 10 kg rod (COM as in the figure, 3 cm from support 1 and 2 cm from support 2, supports 5 cm apart) rests on two supports. Find the normal reactions.

(a) 20 N, 80 N
(b) 30 N, 70 N
(c) 40 N, 60 N
(d) 50 N, 50 N

Answer: (c) — Torque about N_1: $100 \times 3 = N_2 \times 5 \Rightarrow N_2 = 60$ N, $N_1 = 40$ N.

Q10(b). A rod of weight W rests horizontally on knife edges A and B, distance d apart; its COM is at distance x from A. The normal reaction at A is

(a) Wx/d
(b) Wd/x
(c) W(d – x)/x
(d) W(d – x)/d

Answer: (d) — Torque about B: $N_A\, d = W(d - x)$.

Q11. ABC is an equilateral triangle with centre O. Forces F_1, F_2, F_3 act along its sides. If the total torque about O is zero, $|F_3|$ is

(a) $F_1 + F_2$
(b) $F_1 - F_2$
(c) $(F_1 + F_2)/2$
(d) $2(F_1 + F_2)$

Answer: (a) — All sides at the same perpendicular distance d from O: $F_3 d = F_1 d + F_2 d$.

SHIFT OF NORMAL FORCE — TOPPLING

Q12. A block of height h is pushed/projected along a rough floor (coefficient μ). The point of application of the normal force shifts from the centre by x =

(a) h/μg
(b) hμ/g
(c) hμ/2
(d) 0

Answer: (c) — About COM: $f(h/2) = Nx \Rightarrow \mu N(h/2) = Nx$.

Q13. A cube of mass m, edge a slides down a rough incline of angle θ at uniform speed. The torque of the normal force about its centre is

(a) ½ mga sin θ
(b) zero
(c) mga sin θ
(d) mga cos θ

Answer: (a) — Uniform speed ⇒ $f = mg \sin\theta$; $\tau_N = \tau_f = f(a/2) = \frac{1}{2}mga \sin\theta$.

LADDER PROBLEMS

1. **$F_{net} = 0$:** vertical — N_1 (floor) = Mg (+ f_2 if wall is rough); horizontal — N_2 (wall) = f_1 (floor friction).
2. **$\tau_{net} = 0$:** take torque about the foot A (or top B) of the ladder.

Q14. A uniform 10 kg ladder leans against a smooth wall making 53° with it; the foot rests on a rough floor. The normal and frictional forces by the floor are

(a) 100 N, 50 N
(b) 100 N, 66.66 N
(c) 100 N, 133.33 N
(d) 100 N, 33.33 N

Answer: (b) — $N_1 = 100$ N; about A: $100 \times (l/2) \sin 53° = N_2 \times l \cos 53° \Rightarrow 40 = 0.6\, N_2 \Rightarrow N_2 = f = 66.67$ N.

Q15. A uniform 20 kg, 5 m rod leans against a smooth wall making 60° with it; the foot rests on a rough floor. The friction force on the rod is (g = 10)

(a) 100 N
(b) 100√3 N
(c) 200 N
(d) 200√3 N

Answer: (b) — About the foot: $200 \times 2.5 \sin 60° = N_2 \times 5 \cos 60° \Rightarrow N_2 = f = 100\sqrt{3}$ N.

MOMENT OF INERTIA (ADDITIONAL TOPIC)

- **Moment of inertia (I)** — rotational analogue of mass: $\mathbf{I = \Sigma m_i r_i^2}$ (kg m²); depends on mass, its distribution and the axis.
- **Radius of gyration:** $I = MK^2$.

Body	Axis	I
Ring (M, R)	Through centre, ⊥ plane	MR^2
Disc	Through centre, ⊥ plane	$MR^2/2$
Thin rod (L)	Through centre, ⊥ rod	$ML^2/12$
Thin rod	Through one end, ⊥ rod	$ML^2/3$
Solid sphere	Diameter	$(2/5)MR^2$
Hollow sphere	Diameter	$(2/3)MR^2$
Solid cylinder	Own axis	$MR^2/2$

Theorems

Parallel axis: $I = I_{cm} + Md^2$

Perpendicular axis (planar body): $I_z = I_x + I_y$

ROTATIONAL DYNAMICS — ANALOGY (ADDITIONAL TOPIC)

Linear	Rotational
Displacement s	Angle θ
Velocity v	ω (v = rω)
Acceleration a	α (a_t = rα)
Mass m	Moment of inertia I
Force F = ma	Torque τ = Iα
Momentum p = mv	Angular momentum L = Iω = r × p
F = dp/dt	τ = dL/dt
KE = ½mv²	KE = ½Iω²
Power P = Fv	P = τω

- **Conservation of angular momentum:** if $\tau_{ext} = 0$, Iω = constant (e.g. a skater pulls arms in → I ↓ → ω ↑).
- **Rolling without slipping:** $v_{cm} = R\omega$; $KE = \frac{1}{2}mv^2(1 + K^2/R^2)$; acceleration down an incline $a = g \sin\theta/(1 + K^2/R^2)$ — solid sphere reaches first, ring last.

FORMULA SHEET

1. $\tau = r \times F$; $|\tau| = rF \sin\theta = rF_\perp = Fr_\perp$
2. ACW torque +ve, CW –ve; $\tau_{net} = \Sigma\tau$
3. Static equilibrium: $F_{net} = 0$, $\tau_{net} = 0$ about any point
4. Normal-force shift: $x = \mu h/2$
5. Ladder: $N_{floor} = Mg$, $f = N_{wall}$; take torque about the foot
6. $I = \Sigma mr^2$; $\tau = I\alpha$; $L = I\omega$; $KE = \frac{1}{2}I\omega^2$

Torque about the right point = half the question solved!