KSQUARE CAREER INSTITUTE

ELECTRIC POTENTIAL

Class 12 Physics • Potential, Work, Dipole, E–V Relation, Spheres & Shells • NEET / JEE Notes

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Topics Covered

1. Potential difference & potential at a point; unit
2. Potential due to a point charge & system of charges
3. Work done in moving a charge; equipotential surfaces
4. Potential due to a ring and an electric dipole (axial, equatorial, general point)
5. Relation between E and V — E = –dV/dr, partial derivatives
6. Potential due to conducting & non-conducting spheres and concentric shells
7. 20 PYQs solved • Added: potential energy, equipotential properties, capacitance basics — formula sheet

NEET Point / PYQ Alert

Electrostatics = charge & field + **potential** + potential energy (capacitors form a separate topic). Potential is related to **energy** and always needs a **reference** (usually $V_\infty = 0$).

POTENTIAL DIFFERENCE & POTENTIAL

Definitions

$$V_B - V_A = W_{ext\,(A\to B)} / q \quad \text{(charge moved very slowly, no KE change)}$$

$$\Delta V = W_{ext} / q\ ; \quad \text{SI unit: J/C = volt (V)}$$

$$\text{Potential at a point: } V_A = W_{ext\,(\infty\to A)} / q \quad \text{(assuming } V_\infty = 0\text{)}$$

Q1. The potential at A is 100 J/C and 20 J of work has to be done to move 5 C from A to B. The potential at B is

(a) 80 J/C
(b) 104 J/C
(c) 96 J/C
(d) 120 J/C

Answer: (b) — $V_B - V_A = 20/5 = 4 \Rightarrow V_B = 104$ J/C.

▸ Potential due to Point Charges

Point charge & system

$$V = kQ/r \quad \text{(scalar; put Q with its sign)}$$

$$\text{System: } V = V_1 + V_2 + V_3 + \ldots = k(q_1/r_1 + q_2/r_2 + \ldots) \quad \text{(algebraic sum)}$$

Q2. Charges –Q, –q, 2q and 2Q are placed at the corners of a square. For the potential at the centre to be zero **[2012]**

(a) Q = –q
(b) Q = –1/q
(c) Q = q
(d) Q = 1/q

Answer: (a) — Same distance r: –Q – q + 2q + 2Q = 0 ⇒ Q = –q.

Q3. Charges +q, +q, –q, –q are at the corners of a square of side 2L. The potential at A, midway between the two +q charges, is **[2011]**

(a) $(1/4\pi\varepsilon_o)(2q/L)(1 + \sqrt{5})$
(b) $(1/4\pi\varepsilon_o)(2q/L)(1 + 1/\sqrt{5})$
(c) $(1/4\pi\varepsilon_o)(2q/L)(1 - 1/\sqrt{5})$
(d) Zero

Answer: (c) — +q charges at distance L; –q charges at $\sqrt{5}$ L ⇒ V = 2kq/L – 2kq/($\sqrt{5}$L).

WORK DONE IN MOVING A CHARGE

Work

$$W_{ext} = q(V_B - V_A) = q\Delta V \quad \text{(final – initial)}$$

Work depends only on the initial and final positions (conservative field)

Q4. A charge +Q is at the origin. The work done in taking a charge +q from (2a, 0) to (a, 0) is

(a) KQq/a
(b) 3KQq/2a
(c) 2KQq/a
(d) KQq/2a

Answer: (d) — W = qKQ(1/a – 1/2a) = KQq/2a.

Q5. A charge +q is at the origin. The work done in taking –Q from A (0, a) to B (a, 0) along the straight path AB is **[2005]**

(a) Zero
(b) $(qQ/4\pi\varepsilon_o a^2)\sqrt{2}a$
(c) $(-qQ/4\pi\varepsilon_o a^2)\sqrt{2}a$
(d) $(qQ/4\pi\varepsilon_o a^2)(a/\sqrt{2})$

Answer: (a) — A and B are equidistant from the origin ⇒ $V_A = V_B$.

Q6. Three charges +q are at the corners of an isosceles triangle ABC with BC = AC = 2a; D and E are the mid-points of BC and CA. The work done in taking Q from D to E is **[2011]**

(a) $3qQ/4\pi\varepsilon_o a$
(b) $3qQ/8\pi\varepsilon_o a$

(c) $qQ/4\pi\varepsilon_o a$
(d) Zero

Answer: (d) — By symmetry $V_D = V_E$.

Q7. +q at A and –q at B are 2L apart; C is the mid-point. The work done in moving +Q along the semicircle CRD (radius L, centre B) from C to D is **[2007]**

(a) $qQ/2\pi\varepsilon_o L$
(b) $qQ/6\pi\varepsilon_o L$
(c) $-qQ/6\pi\varepsilon_o L$
(d) $qQ/4\pi\varepsilon_o L$

Answer: (c) — $V_C = 0$; $V_D = kq/3L - kq/L = -2kq/3L \Rightarrow W = Q(V_D - V_C) = -qQ/6\pi\varepsilon_o L$.

Q8. Four diagrams show equipotential regions (10 V → 30 V or 20 V → 40 V). A positive charge is moved from A to B in each. Then

(a) maximum work in figure (c)
(b) work done is the same in all four cases
(c) minimum work in figure (a)
(d) maximum work in figure (b)

Answer: (b) — $\Delta V = 20$ V in every case $\Rightarrow W = q\Delta V$ is the same.

POTENTIAL DUE TO A RING

Ring (charge Q, radius R), on axis at distance x

$$V = kQ / \sqrt{(x^2 + R^2)}\ ; \quad \text{at centre } V = kQ/R$$

$$\text{Field: } E = kQx/(x^2 + R^2)^{3/2}\ ; \quad E_{centre} = 0$$

NEET Point / PYQ Alert

If **E = 0**, V is **not necessarily zero** (centre of ring). If **V = 0**, E is **not necessarily zero** (equatorial line of dipole).

POTENTIAL DUE TO AN ELECTRIC DIPOLE

Position	Potential
Axial line (distance r from centre)	$V = kp/(r^2 - l^2)$; short dipole ($r \gg l$): $\mathbf{V = kp/r^2}$
Equatorial line	**V = 0** (but $E \neq 0$)
General point (r, θ) — short dipole	$\mathbf{V = kp\cos\theta / r^2}$

Q9. A short dipole has $p = 16 \times 10^{-9}$ C m. The potential at 0.6 m from its centre on a line making 60° with the axis is ($1/4\pi\varepsilon_o = 9 \times 10^9$)

(a) 400 V
(b) Zero
(c) 50 V
(d) 200 V

Answer: (d) — $V = 9\times10^9 \times 16\times10^{-9} \times 0.5 / 0.36 = 200$ V.

RELATION BETWEEN E AND V

- **Electric potential always decreases in the direction of the electric field** (e.g. for +Q: kQ/1 > kQ/2 > kQ/3); it increases opposite to E.

Relations

$$\Delta V = -\,E \cdot \Delta r\ ; \quad |\Delta V| = E\,\Delta r \cos\theta \ \text{(uniform field)}$$

$$V \text{ given} \Rightarrow E:\ E = -dV/dr\ ; \quad E_x = -\partial V/\partial x,\ E_y = -\partial V/\partial y,\ E_z = -\partial V/\partial z$$

$$E \text{ given} \Rightarrow V:\ dV = -E \cdot dr \Rightarrow V = -\int E \cdot dr$$

Partial derivative: differentiate w.r.t. one variable, treat others as constants

Q10. $\vec{E} = 50\hat{i} + 75\hat{j}$ N/C exists in a region. With V = 0 at the origin, the potential at P(1 m, 2 m) is

(a) 100 V
(b) –100 V
(c) 200 V
(d) –200 V

Answer: (d) — $\Delta V = -E \cdot r = -(50 \times 1 + 75 \times 2) = -200$ V.

Q11. In a uniform field E = 10 N/C (to the right), A, B, C form an equilateral triangle of side 2 m with BC along E (B on the left, A at the apex). Find (a) $V_A - V_B$ (b) $V_B - V_C$.

Answer: (a) –10 V (b) +20 V — $V_A - V_B = -E(2\cos 60°) = -10$ V; $V_B - V_C = +E \times 2 = +20$ V (B is upstream of C).

Q12. A, B and C are three points in a uniform field (B furthest upstream, then C, then A along E). The potential is **[2013]**

(a) maximum at A
(b) maximum at B
(c) maximum at C
(d) same at A, B, C

Answer: (b) — V decreases along E: $V_B > V_C > V_A$.

Q13. $V = 4x^2$ volt. The electric field at (1, 0, 2) is

(a) 8 V/m along –x
(b) 8 V/m along +x
(c) 16 V/m along –x
(d) 16 V/m along +z

Answer: (a) — $E_x = -8x = -8$ at x = 1.

Q14. $V = -x^2y - xz^3 + 4$. The electric field is **[2009]**

(a) $\hat{i}2xy + \hat{j}(x^2 + y^2) + \hat{k}(3xz - y^2)$
(b) $\hat{i}z^3 + \hat{j}xyz + \hat{k}z^2$
(c) $\hat{i}(2xy - z^3) + \hat{j}xy^2 + \hat{k}3z^2x$
(d) $\hat{i}(2xy + z^3) + \hat{j}x^2 + \hat{k}3xz^2$

Answer: (d) — $E_x = 2xy + z^3$, $E_y = x^2$, $E_z = 3xz^2$.

Q15. $V = 6xy - y + 2yz$. The electric field at (1, 1, 0) is **[2015]**

(a) $-(6\hat{i} + 9\hat{j} + \hat{k})$
(b) $-(3\hat{i} + 5\hat{j} + 3\hat{k})$
(c) $-(6\hat{i} + 5\hat{j} + 2\hat{k})$

(d) $-(2\hat{i} + 3\hat{j} + \hat{k})$

Answer: (c) — $E = -(6y, 6x - 1 + 2z, 2y) = -(6, 5, 2)$.

Q16. $V = 6x - 8xy - 8y + 6yz$. The force on a 2 C charge at (1, 1, 1) is

(a) $6\sqrt{5}$ N
(b) 30 N
(c) 24 N
(d) $4\sqrt{35}$ N

Answer: (d) — $E = (2, 10, -6) \Rightarrow |E| = \sqrt{140} = 2\sqrt{35} \Rightarrow F = qE = 4\sqrt{35}$ N.

POTENTIAL DUE TO CHARGED SPHERES & SHELLS

▸ Conducting Sphere (solid or hollow) — charge Q, radius R

Region	Potential	Field
Outside (r > R)	kQ/r	kQ/r^2
Surface (r = R)	kQ/R	kQ/R^2
Inside (r < R)	**kQ/R (same as surface — constant)**	**0**

Q17. A conducting sphere of radius R has charge Q. The potential and field at its centre are **[2014]**

(a) zero and $Q/4\pi\varepsilon_0R^2$
(b) $Q/4\pi\varepsilon_0R$ and zero
(c) $Q/4\pi\varepsilon_0R$ and $Q/4\pi\varepsilon_0R^2$
(d) both zero

Answer: (b)

▸ Concentric Shells

- Potential at a point = Σ (potential due to each shell): for a shell of charge q and radius R — **kq/r if the point is outside, kq/R if inside**.

Q18. Two concentric conducting shells of radii $r_1 < r_2$ carry q_1 and q_2. Find the potential at distance r for (i) $r < r_1$ (ii) $r_1 < r < r_2$ (iii) $r > r_2$.

Answer: (i) $k(q_1/r_1 + q_2/r_2)$ (ii) $k(q_1/r + q_2/r_2)$ (iii) $k(q_1 + q_2)/r$

Q19. Concentric metal shells A, B, C (radii a < b < c) have surface charge densities +σ, −σ, +σ. The potential of shell B is

(a) $(\sigma/\varepsilon_0)[(b^2 - c^2)/c + a]$
(b) $(\sigma/\varepsilon_0)[(a^2 - b^2)/a + c]$
(c) $(\sigma/\varepsilon_0)[(a^2 - b^2)/b + c]$
(d) $(\sigma/\varepsilon_0)[(b^2 - c^2)/b + a]$

Answer: (c) — $q = \sigma 4\pi R^2$: $V_B = (\sigma/\varepsilon_0)(a^2/b - b + c)$.

Q20. Three concentric shells (radii a < b < c) have charge densities σ, −σ, σ. If c = a + b, then **[2009]**

(a) $V_C = V_B \neq V_A$
(b) $V_C \neq V_B \neq V_A$
(c) $V_C = V_B = V_A$
(d) $V_C = V_A \neq V_B$

Answer: (d) — $V_A = (\sigma/\varepsilon_0)(a - b + c)$; $V_C = (\sigma/\varepsilon_0)[(a^2 - b^2)/c + c] = (\sigma/\varepsilon_0)(a - b + c)$ when c = a + b.

▸ Non-conducting Solid Sphere (uniform charge Q)

Region	Potential
Outside (r > R)	kQ/r
Surface (r = R)	kQ/R
Inside (r < R)	$kQ(3R^2 - r^2)/(2R^3)$
Centre (r = 0)	**(3/2) kQ/R = 1.5 $V_{surface}$**

BEYOND THE LECTURE (ADDITIONAL TOPIC)

Topic	Key formula / fact
Potential energy of two charges	$U = kq_1q_2/r$; system: $U = \Sigma (kq_iq_j/r_{ij})$ over all pairs
PE of dipole in uniform E	$U = -p \cdot E = -pE \cos\theta$
Equipotential surfaces	Same potential everywhere; **no work** in moving a charge on them; E is **perpendicular** to them; never intersect; closely spaced where field is strong
Conductors	E = 0 inside; surface is equipotential; charge resides on the outer surface; field just outside = σ/ε_o (normal to surface)
Capacitance	$C = Q/V$; parallel plate $C = \varepsilon_oA/d$; with dielectric $C = K\varepsilon_oA/d$; energy $U = \frac{1}{2}CV^2 = Q^2/2C$
Combination of capacitors	Series $1/C = \Sigma 1/C_i$; parallel $C = \Sigma C_i$

FORMULA SHEET

1. $\Delta V = W/q$; $V = kQ/r$; $V_{system} = \Sigma kq_i/r_i$
2. $W = q\Delta V$ (path independent)
3. Ring: $V = kQ/\sqrt{(x^2 + R^2)}$; centre kQ/R
4. Dipole: $V_{axial} = kp/r^2$, $V_{equatorial} = 0$, $V(r, \theta) = kp \cos\theta/r^2$
5. $E = -dV/dr$; $E_x = -\partial V/\partial x$ …; $\Delta V = -E \cdot \Delta r$
6. Conducting sphere: $V_{in} = kQ/R$; $E_{in} = 0$
7. Non-conducting sphere: $V_{in} = kQ(3R^2 - r^2)/2R^3$; $V_{centre} = 1.5\ kQ/R$

V decreases along E — remember it in every question!